An Approximate Formula for Calculating Lateral Displacements of Sway Steel Structures with infill panels

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Abstract:

Most international codes require the calculation of lateral displacements of sway frames for the sake of comparing these displacements with the allowed displacements or for the sake of calculating the nonlinear effects of these frames through calculating the elastic critical factor of steel frames.

A new model has been developed in this paper to calculate the lateral displacement of steel frames taking into account the beam and column stiffness of these frames together with the stiffening effects resulted from the presence of infill panels (if required).

To justify the accuracy of the proposed method to handle practical frames two examples have been calculated utilizing the proposed method and with the well-known structural software SAP 2000 and a comparison of the obtained results has shown that the accuracy of the proposed method is appropriate for design use.

Since the proposed method is simple and easy to be used and requires only minimum effort, it is ideally suited for design use.

Keywords: Lateral Displacements, Sway Framed structures, Approximate method.

List of Symbols:

dm: diagonal length of infill panel.

E: Young's modulus.

G_i: ratio of the sum of beam rotational stiffnesses to the sum of the total story rotational stiffness (beams plus columns) of the story i.

h: story height.

H_i: lateral load applied at story i.

L_b: beam span.

L_c: column height.

I_b: second moment of area of the beam.

I_c: second moment of area of the column.

$\bar{S}_i$: non-dimensional parameter accounts for the strengthening effect resulted from the presence of infill panels in practical buildings.

t: thickness of infill panel.

W: width of infill panel.

β: diagonal angle of infill panel.

δ_i: lateral displacement of story i.

θ: rotation at upper and lower column ends of the model.

Introduction

Most international codes require the calculation of lateral displacements of steel frame with infill panels because these panels reduce the lateral displacements. This issue is usually calculated by using finite element software as (SAP 2000). Previous studies lacked simple mathematical equations for calculating such as these displacements. Therefore, this research aims to develop an easy method by finding simple mathematical

equation to calculate the lateral displacement of steel frames with infill panels without resorting to finite element software.

Proposed Model

In order to calculate an approximation to lateral deflection of sway frames, the model shown in figure 1 is adopted. This model consists of a column connected to upper and lower beams, which, for the sake of simplicity, are assumed to be similar.

This model has also a linear lateral spring S_i which allows for, if required, the stiffening effects resulted from the presence of infill panels in story i in real buildings, i.e. brick walls.

Calculating the end moments of the model assuming positive anti-clock wise gives

$$M_{21} = M_{12} = \frac{EI_c}{L_c}\left[60 + \frac{6\delta_i}{L_c}\right] \;\; ; \;\; M_{2B} = \frac{EI_c}{L_b}[30] \qquad (1)$$

Where:

E: Young's modulus.

I_c: second moment of area of beam.

I_b: second moment of area of column.

θ: rotation at upper and lower column ends equilibrium at joint 2 requires that

H_i: is lateral load applied at story i.

L_b: is beam span.

L_c: is column height.

δ_i: story drift.

$$\sum_2 M_{21} + M_{12} = 0 \Rightarrow \left[\frac{6EI_c}{L_c} + \frac{3EI_b}{L_b}\right]\theta + \frac{6EI_c}{L_c}\,\frac{\delta_i}{L_c} = 0 \;\Rightarrow\; \theta = \left(\frac{-\frac{6EI_c}{L_c}\frac{\delta_i}{L_c}}{\frac{6EI_c}{L_c}+\frac{3EI_b}{L_b}}\right) \qquad (2)$$

Considering the equilibrium of the column one gets:

$$\sum_{column} M = 0 \;\Rightarrow M_{21} + M_{12} + (S_i.\delta_i - H_i)L_c = 0 \qquad (3)$$

Substituting equations 1 and 2 and accomplishing great amount of manipulation gives:

$$\delta = \frac{\frac{HL_c{}^3}{12EI_c}}{G_i + \bar{S}_i}$$

Where:

$$\bar{S}_i = \frac{S_i L_c{}^3}{12EI_c}$$

And,

$$G_i = \frac{\frac{3EI_b}{L_b}}{\frac{3EI_b}{L_b} + \frac{6EI_c}{L_c}} \qquad (4)$$

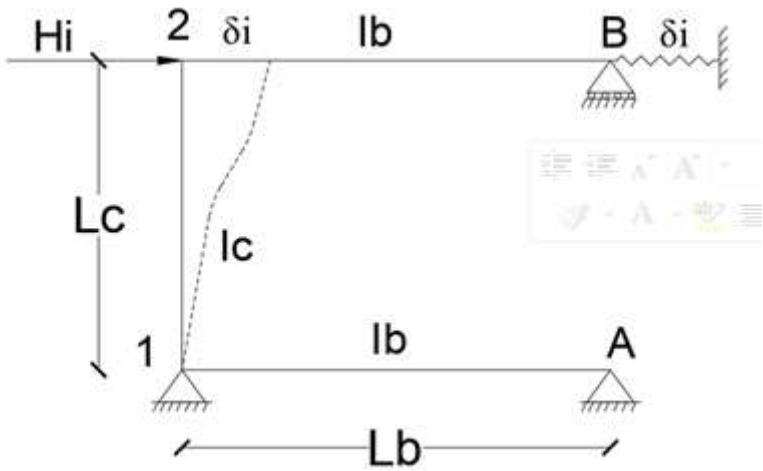

Figure 1: Proposed model

In order to use equation 4 for a multistory frame, it should consider the following points:

- In practical sway frames, each beam buckles in double curvature. Hence the rotational stiffness of the beam should be multiplied by 2.
- In practical frames, each beam restrains two columns. Therefore, the beam rotational stiffness should be multiplied by 2.
- Finally, each beam in a practical multistory sway frame restrains two columns, i.e. upper and lower columns connected to that beam. Thus the beam rotational stiffness should approximately be divided by 2.

As a result of above the use of equation 4 for practical multistory frame is permitted providing that the beam rotational stiffness is multiplied by 2, or

$$G_i = \frac{\sum_{story} \frac{6EI_b}{L_b}}{\sum_{story} \frac{6EI_b}{L_b} + \frac{6EI_c}{L_c}}$$

$$G_i = \frac{\sum_{story} \frac{I_b}{L_b}}{\sum_{story} \frac{I_b}{L_b} + \frac{I_c}{L_c}} \qquad (5)$$

It should be noted here that equation 5 underestimates the stiffness of both first story, where columns might be fully fixed at base, and for upper story, where no upper columns are present. This underestimation should not affect the accuracy of equation 4 as will be seen later in the worked examples.

The factor G_i should be taken as the ratio of the sum of beam rotational stiffnesses of the i^{th}story to the sum of the total story rotational stiffness (beams plus columns).

Corresponding to equation 4 the i^{th}story drift resulted from subjecting that story to horizontal load H_{ei} equal to the sum of horizontal loads applied at the

story level and above, i.e. $H_{ei} = \sum_{i=i}^{i=n} H_i$:

$$\delta_i = \frac{H_{ei}}{12E}\frac{{L_{ci}}^3}{I_{ci}}\left(\frac{1}{G_i + \bar{S}_i}\right) \qquad (6)$$

Where:

I_{ci}: sum of the second moment of areas of beams of i^{th}story.

L_{ci}: height of i^{th} story.

G_i : the ratio of beam stiffness to column and beam stiffness of the i^{th} story (equation 5).

Since equation 5 calculate the relative lateral displacement of the story therefore the previous story displacements should be added to it (see the worked examples).

Calculating the factor $\bar{S}_i$ for practical buildings

The non-dimensional factor $\bar{S}_i$ which allows for the strengthening effect resulted from the presence of infill panels in practical buildings, can be $calculated^{(5)}$ as follows:

Enforced horizontal displacement at the top of i^{th} story is considered (1) (see figure 2), the corresponding diagonal displacement of the story equal to (1. cos β) and the diagonal force on the struts becomes equal to:

$$\text{diagonal force} = \frac{1\cos\beta}{dm} Ewt \qquad (7)$$

Where:

β: panel diagonal angle (see figure 2) E: Young's modulus of the panel material.

w: the width of the panel

t: the thickness of the panel.

dm: the diagonal length of the panel hence the horizontal force equal to:

$$\text{horizontal force} = \frac{1\cos\beta^2}{dm} Ewt \qquad (8)$$

This is equal to the stiffness S, since S is calculated by:

$$S = \frac{\text{horizontal force}}{\text{horizontal displacement}}$$

The panel width w depends on the diagonal length of the panel dm and can be taken for practical buildings as w=0.25*dm [3] substituting this relation into equation 8 gives:

$$S = 0.25 * (\cos\beta)^2 * E * t \qquad (9)$$

S_i should be taken as the sum of S for all infill panels of the i^{th}story, and,

$$\bar{S_i} = \frac{S_i}{12\, EI_{ci}}$$

Figure 2 shows that accounting for the stiffening effects resulted from the presence of infill panels in practical frames analysis can be accomplished approximately by replacing each infill panel with a diagonal strut element. The cross section of the strut element should be taken as w*t with Young's modulus equals to the Young modulus of the panel materials.

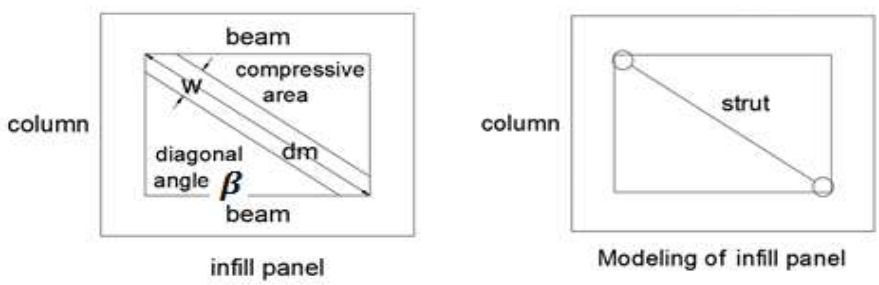

Figure 2: Modeling of infill panel.

Verification of the proposed method

In order to verify the accuracy of the proposed method (equation 6) to calculate a good approximation to the lateral deflection of framed building, the following two examples are selected.

Example 1:

The frame shown in figure (3) which consists of ten multistory two bays steel frame is utilized in example 1. It is assumed that **infill panels are extended over the full height of all bays of the frame** with the following properties for infill panel:E=1000 MPa, t=10 cm, the effective width of the compression region equals to 0.25 of the diagonal length of the corresponding bay. Moreover, Young's modulus for steel is taken equaled to 210000 MPa.

The example includes studying one frame in direction x, because the results are similar between x and y directions, this matter is attributed to the symmetry of the building, and all the joints in the frame model are rigid joints.

The effect of vertical loads in calculating lateral displacements in steel structures is much smaller than the lateral loads. Therefore, this effect is neglected.

Calculating story properties for the first story of the frame produces:

$$G_i = \frac{\sum_{story} \frac{I_b}{L_b}}{\sum_{story} \frac{I_b}{L_b} + \frac{I_c}{L_c}}$$

$$G_1 = \frac{2 * \frac{5115}{600} + \frac{5115}{400}}{2 * \frac{5115}{600} + \frac{5115}{400} + \frac{11720}{300}} = 0.43$$

$$S = 0.25 * (\cos\beta)^2 * E * t$$

$$S_1 = (0.25 * 0.8^2 * 100 * 10) = 160\ KN/cm$$

$$\bar{S}_1 = \frac{160 * 300^3}{12 * 21000 * 11720} = 1.46$$

$$\delta_i = \frac{H_{ei}}{12E} \frac{{L_{ci}}^3}{I_{ci}} \left(\frac{1}{G_i + \bar{S}_i} \right)$$

It should be noted that the second moment of inertia of the beams and columns in the model are supposed values in SAP 2000 regardless of the sections of these elements (beams, columns), and the infill panel is modeled as strut when utilizing SAP 2000 to calculate the lateral displacement of the frame with infill panels.

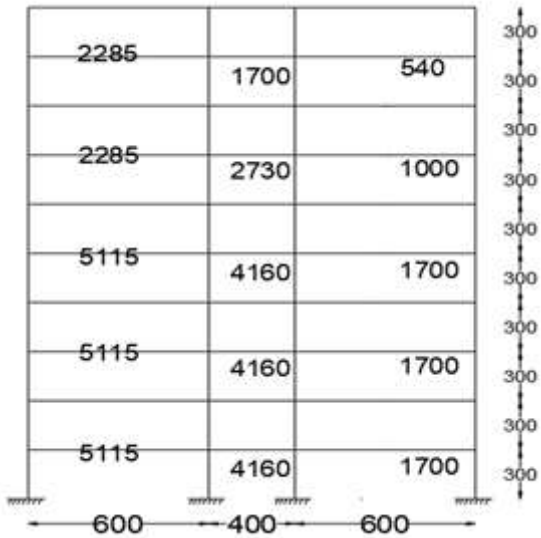

Figure 3: The frame of example 1.

Second moment of area for each element is shown in table 1.

Table 1: Values of second moment of inertia of all elements.

Story	$I_{c\ (exterior)}$ (cm4)	$I_{c\ (interior)}$ (cm4)	I_b (cm4)
1,2,3,4,5,6	1700	4160	5115
7,8	1000	2730	2285
9,10	540	1700	2285

Horizontal loads applied at each story level equal to 30 kN and only on the top story is taken equaled to 20 kN.

It should be noted here that the effective width of the infill panels w is taken equal to 125 cm for all spans of the frame (400 cm, 600 cm).

Table 2 shows values of lateral displacements for each story in the studied frame.

Table 1: Comparison between lateral displacements of proposed formula and SAP 2000 of example 1.

Story	$I_{ci}\sum_{story} I_c$	G_i	H_{ei} (KN)	$\bar{S}_i$	$G+\bar{S}$	δ_i (cm)	Proposed δ_i,t (cm)	SAP2000 δ_i,t (cm)
1	11720	0.43	285	1.46	1.89	1.38	1.38	1.40
2	11720	0.43	255	1.46	1.89	1.23	2.61	2.65
3	11720	0.43	225	1.46	1.89	1.09	3.70	3.72
4	11720	0.43	195	1.46	1.89	0.94	4.64	4.60
5	11720	0.43	165	1.46	1.89	0.80	5.44	5.40
6	11720	0.43	135	1.46	1.89	0.65	6.09	6.00
7	7460	0.35	105	2.29	2.64	0.57	6.66	6.60
8	7460	0.35	75	2.29	2.64	0.41	7.07	7.00
9	4480	0.47	45	3.82	4.29	0.25	7.32	7.25
10	4480	0.47	15	3.82	4.29	0.08	7.40	7.30

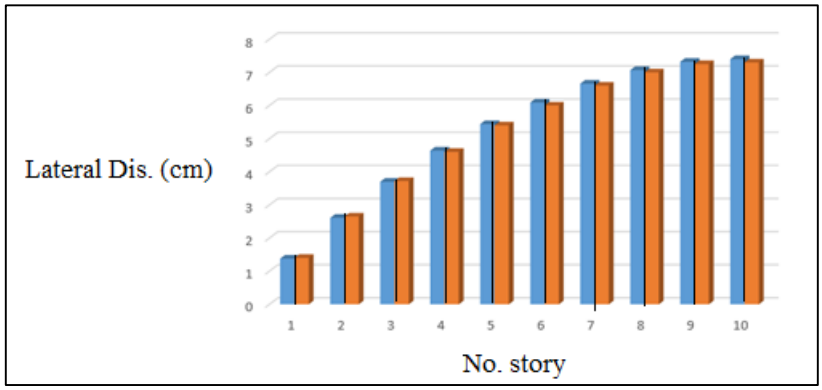

Figure 4: Comparison between lateral displacements of proposed formula (left columns) and SAP 2000 (right columns) of example 1.

As shown in Table. 1, there is a good agreement between the values resulted in the proposed approximate formula and the analytical values resulted in SAP 2000 software, confirming the accuracy of the proposed formula in current paper.

Example 2:

Same procedure used in example 1 is repeated here for the frame shown in figure (6) which consists of three stores one bay frame. The result is tabulated in table 2.

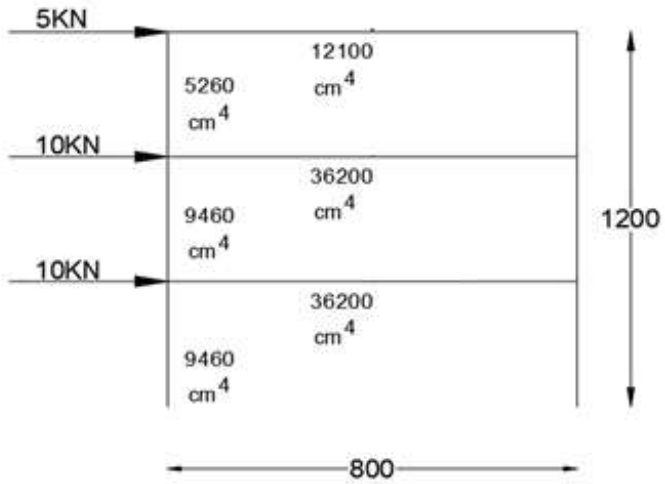

Figure 5: The frame of example 2.

Table 2: Comparison between lateral displacements of proposed formula and SAP 2000 of example 2.

story	$I_{ci}\sum_{story} I_c$	G_i	H_{ei} (KN)	δ_i (cm)	Proposed δ_i,t (cm)	SAP2000 δ_i,t (cm)
1	18920	0.25	25	1.34	1.34	1.31
2	18920	0.49	15	0.41	1.75	1.71

3	10520	0.365	5	0.33	2.09	2.02

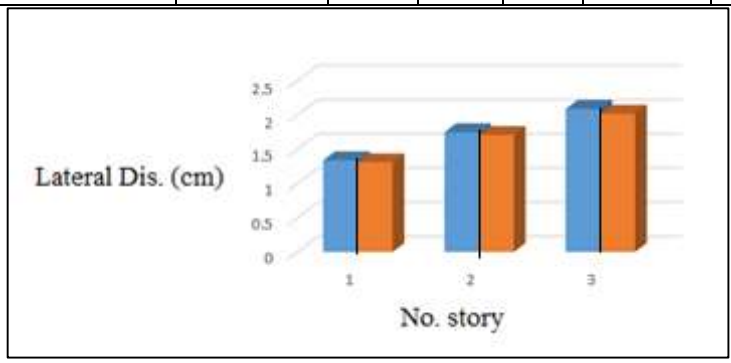

Figure 6: Comparison between lateral displacements of proposed formula (left columns) and SAP 2000 (right columns) of example 2.

As shown in Table. 2, there is again a good correlation between the proposed model and the exact analytical values employing SAP 2000 software.

Conclusion

A simple design method to calculate the lateral displacements of steel frame buildings has been proposed. The proposed method is simple to use and provides a reasonably accurate estimation of lateral displacements of steel framed structures. It takes into consideration the presence of infill panels in approximate and practical way.

Two examples have been analyzed and solved by the author utilizing the proposed method and SAP 2000 software. Comparisons of the results obtained have shown clearly that the proposed method provides satisfactory accurate estimations of lateral displacements of practical steel frame buildings. Since the proposed method requires only minimum hand calculation, it is believed to be ideal for design office use.

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استخدام معادلة تقريبية لحساب الانزياحات الأفقية للمنشآت المعدنية (القابلة للانزياح) مع وجود جدران مالئة

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الملخص

تتطلب معظم الكودات العالمية حساب الانتقالات الأفقية للإطارات المعدنية التي تخضع لانزياح وذلك بغية مقارنتها مع الانتقالات المسموحة أو بغية استعمالها لحساب السلوكية اللاخطية لهذه الإطارات وذلك بعد حساب ما يسمى بمعامل التحنيب المرن لهذه الإطارات.

لقد تم في هذا البحث تطوير موديل جديد لحساب الانتقالات الجانبية للمنشآت المعدنية آخذين بعين الاعتبار صلابة أعمدة وجيزان هذه المنشآت. وقد تم أيضا في هذا المقال الأخذ بعين الاعتبار (عند الرغبة) الصلابات الجزئية التي تتوفر للمنشآت المعدنية من وجود القواطع القرميدية لما لها من أثار إيجابية في تخفيض الانتقالات الجانبية للمنشآت العملية

للتحقق من دقة الطريقة المقترحة تم حل مثاليين باستخدام الطريقة المقترحة في هذه المقالة والبرنامج الإنشائي Sap2000 وتم مقارنة النتائج الحاصلة، حيث أظهرت هذه المقارنة أن الطريقة المقترحة تعطي أجوبة ذات دقة مقبولة لأعمال التصميم.

باعتبار أن الطريقة المقترحة في هذا البحث بسيطة وسهلة وتحتاج إلى عمليات حسابية يدوية قليلة لذلك تعتبر هذه الطريقة مثالية لأعمال التصميم.

الكلمات المفتاحية: إطارات تخضع لانزياح جانبي – انتقالات جانبية – طريقة تقريبية